Background:

The world as we know it is becoming increasingly connected, which in recent years has led to an explosion of effective tools for the analysis of complex networks, aiding in tasks such as clustering, ranking, visualization, describing collective behavior, and so on. Even without getting into the theory, one can appreciate the pervasiveness of network science in our daily lives: if you’ve “Googled” something today, then you’ve (1) utilized a Web search ranking algorithm, (2) accessed the Internet, a global system of computer networks which links devices according to the Internet protocol suite (TCP/IP), (3) operated a device which draws energy from your local power grid, thanks to careful reliability planning on the part of your transmission system operator, and so on. Additionally, network science has been used to study such varied systems as metabolic pathways, opinion dynamics, bird flocking, and more.

How can researchers in mechanical engineering exchange tools with the experts in complex networks? The key idea is this: any dynamical system may be viewed as a network, and vice versa. Below, I provide an example which illustrates how the two are related. The consequence of this connection is that we have the potential to use data-driven techniques from dynamical systems—including those which identify invariant sets, coherent sets, and other features directly from observations of the system—in network science applications. Some of these connections are intuitive: for example, an invariant set in the dynamical system is equivalent to congestion in the network. Therefore, if an efficient algorithm were developed to compute the invariant sets of the dynamical system, then an improved partitioning algorithm would immediately be discovered; indeed, partitioning computations are currently prohibitive for large networks.

Initially, the scope of this work will cover applications of modern data-driven methods in dynamical systems to discover properties related to the structure of and information transfer within complex networks. I will focus on methods such as variants of Dynamic Mode Decomposition (DMD), which is a powerful modal decomposition technique originally introduced by the fluid mechanics community (see brief description below). I will also explore the application of nonlinear model reduction techniques, such as diffusion maps, to the reduction of complex networks. Additionally, I will investigate the application of these methods to create improved algorithms in the field of machine learning. Manifold learning techniques are widely used in the computer vision community for purposes such as human pose estimation, and I plan to explore how these architectures, as well as data augmentation techniques, may benefit from methods from the modern controls and dynamics toolbox.

Dynamical system to network representation:

Let us now consider a classic example of a discrete-time dynamical system [1], to demonstrate how we might represent such a system as a network. The system we consider here is the Baker’s transformation on the unit square, according to:

\[
T(x_1, x_2) = \begin{cases} 
(2x_1, \frac{1}{2}x_2), & x_1 \in [0, \frac{1}{2}) \\
(2x_1 - 1, \frac{1}{2}x_2 + \frac{1}{2}), & x_1 \in [\frac{1}{2}, 1),
\end{cases}
\]

which, as shown here, maps the shaded area at the left into the shaded area at the right, and maps vertical lines to vertical lines.
To represent this as a graph, we may split the unit square into four equal sections, as shown below. Then, an adjacency graph is constructed according to how subsets map to the next respective subset. The graph representation is also shown here, and its corresponding adjacency matrix is binary:

\[
\begin{array}{cc}
1 & 3 \\
2 & 4
\end{array}
\]

For more general dynamical systems, one may refine subsets of the phase space into smaller boxes in regions of interest. Software packages such as Gaio [2]- a toolbox for set-oriented numerics in dynamical systems- use adaptive methods such as this to approximate the Perron-Frobenius operator of a system.

**Dynamic Mode Decomposition (DMD):**

*DMD* is a data-driven (equation-free!) architecture that reconstructs nonlinear dynamics from snapshot measurements of a system alone. DMD in particular is a *modal decomposition technique*, which allows the user to reconstruct DMD modes which are typically more physically meaningful than POD modes due to the fact that they are numerical approximations of the Koopman modes of the underlying system. Because DMD and extensions of it are relatively efficient, quick to implement and act directly on observations of a system, they have become widely applied in the fluid dynamics community to study different flow configurations. Given the close ties between dynamical systems and complex networks, I am confident that DMD can be effectively applied to study the structure and dynamics of networks as well.

**A motivating application in network science:**

Last year, DMD was successfully applied to estimate topological properties of a network of interconnected dynamical systems, published as: “Spectral identification of networks using sparse measurements” [4]. Prior to this work, most network identification methods in the framework of dynamical systems theory were impractical for real networks: all required observations of every node of the network, and many involved invasive modification of the network dynamics or connectivity. With the use of DMD, *global network properties were recovered by tracking just one or a few nodes*. I have already demonstrated that by observing all of the nodes of the network, modal information can be recovered (i.e. congestion information), providing strong incentive for extending results further.

**A potential avenue for improvement in computer vision:**

The field of control systems engineering developed rapidly following the advent of computers. As the field of deep learning matures, we have begun to notice methods from controls echoed in the architectures of certain deep networks: for instance, the predictive-corrective networks developed for web-scale video understanding are analogous to Kalman filters. I intend to explore new applications of controls techniques to computer vision tasks, and to resolve misunderstandings in the computer vision community regarding manifold learning techniques, e.g. the implementation of diffusion maps in [5].

**References:**