Heterogeneity and Synchronization of Coupled Neuronal Oscillator Networks

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The study of synchronization in oscillator networks has the potential to improve treatment of synchronization-based neural disorders, including epilepsy and Parkinson’s disease. With this aim, application of control theory to synchronized neuronal network models has produced a method for desynchronization in the special case of a fully connected network [1]. Here, we investigate synchronization in networks of neuronal oscillator models, with a focus on heterogeneity in external inputs and general network structure that can be used to generalize such control schemes to more realistic contexts.

We investigate synchronous firing patterns in neuronal oscillator networks with gap junction coupling by applying a passivity-based Lyapunov analysis to undirected networks of homogeneous FitzHugh-Nagumo (FN) oscillators. The most commonly analyzed model in network studies is the Kuramoto model [2], but the FN model is higher-dimensional and captures a wider range of dynamics. Our analysis gives sufficient conditions on coupling strength for complete synchronization that are tighter than conditions derived from alternative methods [3, 4]. We extend this analysis to a FN network with differing external inputs wherein cluster synchronization (CS), synchronization of distinct groups of neurons determined by a network partitioning [5], emerges under conditions on the symmetry of the coupling matrix and external inputs. The next step in this line of inquiry is to incorporate complementary methods to find necessary synchronization conditions [6, 7] that provide precise bounds for when synchronization will occur.

Despite an abundance of literature defining CS, utilizing synchronized groups to reduce large sets of coupled neurons to a simple representative network is under-explored [8, 9]. We propose a method to reduce complex oscillator networks to a set of representative oscillators that replicate the full dynamics. We implement this reduction in a completely connected network of oscillators with different input parameters in high-coupling and low-coupling regimes using singular perturbation and a nonsmooth Lyapunov argument. While the high-coupling limit produces behavior equivalent to a single oscillator, lower coupling captures cluster-based state differences. We perform a detailed analysis of the dynamics of two heterogeneous coupled FN oscillators as an illustration of the utility of the reduction. Potential applications of this approach include investigating relationships between neuronal and large-scale brain activity.

Future directions include an extension of the analysis of two-oscillator dynamics to examine how multiple sources of heterogeneity impact the set of quiescent neurons, symmetry-based arguments to simplify the reduction and synchronization condition for multiple type...
of heterogeneity, and incorporation of more sophisticated tools to study synchronization such as contraction theory.

References


