# Flexible Task Allocation Dynamics for Multiple Agents

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## I. OVERVIEW

The overarching goal of my work is to design dynamics from which we can develop systematic methodology for allocating interconnected agents within a multi-agent system across multiple options, or tasks, in a distributed manner. This is motivated by questions of how networked vehicles in remote contexts can be controlled, or can control themselves, in order to carry out tasks such as observation of dynamic environments, tracking of wildlife, or measurements of pollutants in ocean waters. The dynamics I design will synthesize a wide variety of stable group behaviors including agreement within a network, disagreement by parts of the network, and mixed outcomes in between. Furthermore, smooth transitions between the various stable behaviors will be governed by a small number of well understood parameters.

Within multi-agent systems individual agents sense their local environment, receive communicated information about the states of other nearby agents, and update their behavior appropriately in response to these inputs. In our distributed design framework, each agent in a multi-agent system must independently make the choice of which action to take, which direction to go, or which alternative to favor, based on the information it gathers from its environment and from its communication with other nearby agents in the network. At the level of the group the individual choices must add up to a well-coordinated collective behavior.

A flexible mechanism for distributed control of emergent collective agreement for a group of agents choosing between two equally valuable options was the subject of past work carried out in the group [2], [3]. Many design tasks such as resource allocation and autonomous exploration may require group level outcomes beyond consensus, such as subpopulations of agents choosing to carry out different tasks, in order for the group to be successful in its objective. Furthermore within complex environments, change in environmental conditions can change the optimality of the group-level strategy currently being carried out by the system. To address this we must design adaptable systems that are able to smoothly transition from one group-level strategy such as agreement to other complex strategies, such as different tasks being carried out by different subpopulations of agents, in response to external triggers.

Looking to known natural systems to guide modeling provides a logical starting point for our design task. Systems in nature exhibit emergent group behaviors that are stable, robust, and adaptive to change - qualities we do not typically capture together in engineered systems. Two particular models that will serve as my starting point are the Hopfield model for the graded computations performed by networks of two-state neurons [5] and the Replicator-Mutator model which is used to describe the dynamics of complex adaptive systems in population genetics, biochemistry and evolution of language [6]. Both are nonlinear models that feature a multitude of complex emergent behaviors at the group level.

I will begin by considering idealized symmetric versions of the natural models. In multi-agent decision making and task allocation problems symmetries arise naturally and serve as a maximally flexible starting point. One symmetry is associated with permuting individuals in a homogeneous group, and another with permutations among options that are viewed as equally valuable by the agents. The models must then be modified to

1) exhibit the richest possible dynamics
2) have system variables and steady-state solutions that are interpretable in the context of multi-agent task allocation
3) have a minimal number of system parameters that govern transitions between the various possible system configurations

These design goals can be achieved using tools of singularity theory, a robust bifurcation theory that rigorously classifies behavior of dynamical systems near transition points [1]. In the framework of singularity theory we study dynamical systems in one or more state variables whose time evolution is governed by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u) \tag{1}$$

where $\mathbf{f} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a smooth map, $\mathbf{x} \in \mathbb{R}^n$ is the state vector, and $u \in \mathbb{R}$ is a system parameter. Within this formalism, solving the *recognition problem* identifies the dynamics of the system in question with a local equivalence class, the simplest member of which is a representative *normal form*.

The dynamics of normal form equations for systems with permutation symmetries are mapped out in applied mathematics literature. Constraining the models we work with to meet equivalence conditions to known symmetric normal forms provides a direct mapping between the solutions and parameters of the interpretable systems and those of the normal form equations. Thus, we can easily identify and interpret all model behaviors that are available to the nonlinear system, as well as identify the relevant parameters that govern system transitions. Once the models are finalized, I will use feedback to design adaptive protocols for the parameters that enable system response to environmental triggers. This is a novel approach to system design.

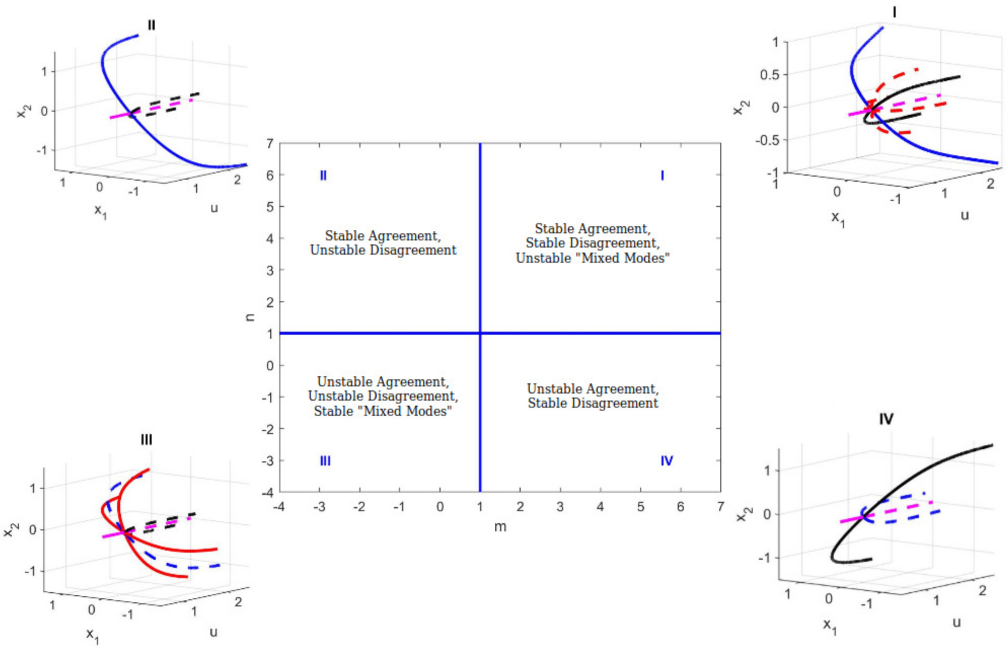


Fig. 1. Four distinct topological configurations the model bifurcation diagram takes on. Solid lines are stable solution branches, dashed lines are unstable solution branches. Solutions in blue lie on the agreement manifold ($x_1 = x_2$, both agents choose the same option with equal magnitude of opinion), solutions in black on the disagreement manifold ($x_1 = -x_2$, agents choose opposing options with equal magnitude of opinion), and solutions in red that appear in two of the four distinct dynamical regions are "mixed" solutions that are steady state outcomes somewhere in between agreement and disagreement.

Realistic systems can have asymmetry due to disturbance and uncertainty encountered in complex environments, which can be modeled as deviation from an idealized symmetric setting. The analysis and design I will carry out using intuition from singularity theory will guarantee robust and stable outcomes when model symmetry is broken. For the symmetric normal forms we consider, the system dynamics can change in a finite number of well understood ways in response to disturbance. Typically we observe that outside of a small neighborhood of the bifurcation point, stable branches of solutions predicted by the symmetric case persist. In other words, symmetry serves as the organizing center for group decision dynamics. It would take a significant amount of disturbance to cause a system in a stable configuration to evolve away from its initial state. Therefore the control protocols I will create will still be reliable in real world asymmetric scenarios.

## II. PRELIMINARY RESULTS

In the beginning stages of my proposed work we consider the simplest scenario: two agents allocating between two equally valuable options or tasks. The results summarized in this section are the subject of a paper in preparation for submission, tentatively titled *Flexible task allocation dynamics for two agents and two options*, and of an upcoming SIAM talk in May. We define $x_i \in \mathbb{R}$ to be the opinion state of agent $i$, with $x_i = 0$ corresponding to agent $i$ being in the state of indifference towards either option, $x_i > 0$ corresponding to a preference for option 1, and $x_i < 0$ corresponding to a preference for option 2.

We start with a symmetric generalization of a Hopfield neural network with two communicating processing units. The dynamics of this two-agent system are organized by an $S_2 \times S_2$-symmetric nondegenerate normal form, the recognition problem for which is worked out in [1]. The finalized model takes the form

$$\dot{x}_i = -x_i + u\Big(S(cx_j) + \frac{1}{2}S(2(x_i - cx_j))\Big) \qquad (2)$$

where $S(\cdot)$ is an odd sigmoid saturating function and $u, c$ are system parameters. $u$ is the bifurcation parameter varying which drives the system from a steady state of group indecision to one in which indecision destabilizes and a number of nontrivial steady states emerge. Varying the parameter $c$ modifies the topological structure of the bifurcation diagram.

There are four distinct parameter space configurations the system can take on, pictured in bifurcation diagrams in Fig. 1. With smooth variation of a single parameter $c$ we can drive the system from a configuration within which consensus is the only stable group behavior to one within which disagreement is the only stable group behavior, among many other potential transitions. Next we will incorporate environmental sensing and feedback dynamics into the control parameters in the model to enable smooth transitions between dynamical behaviors that adapt to environmental triggers. We will then consider more complex cases, starting with 2 agents choosing between 3 options and 3 agents choosing between 2 options in a similar symmetric framework, with further generalizations to larger systems down the line.

## REFERENCES


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