Modeling Turbulent Separation Bubbles with Dynamic Mode Decomposition

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Dynamic Mode Decomposition

Dynamic mode decomposition is a data-driven reduced order modeling technique commonly used for fluids applications, where the system's state $\mathbf{x}(t)$ is broken down into modes with time-varying coefficients,

$$\mathbf{x}(t) = \sum_{j=1}^{N} c_j e^{\lambda_j t} \mathbf{v}_j,$$

where we notice the time-varying coefficients must take the form of exponential functions, leading to simple physical interpretation – modes $\mathbf{v}_j$ are growing, shrinking, and oscillating in time according to their corresponding values of λ_j. In extended dynamic mode decomposition (EDMD), "observable" functions are applied to the data $\mathbf{x}(t)$ before DMD is performed, which allows us to model a wider range of system behavior. In the DMD algorithm, a "DMD matrix" A is identified whose eigenvalues are λ_j and whose corresponding right eigenvectors are $\mathbf{v}_j$.

EDMD is related to a linear, infinite dimensional operator called the Koopman operator. For a system with dynamics $x(k+1) = F(x(k))$, the Koopman operator $\mathcal{U}$ acts on scalar functions g of the state space such that $\mathcal{U}g(x) = g(F(x))$. It has been shown that, with some constraints on the observables chosen, eigenvalues of the DMD matrix A resulting from EDMD are eigenvalues of the Koopman operator $\mathcal{U}$, and the corresponding left eigenvectors of A correspond with the associated eigenfunction of the Koopman operator [1].

Based on this connection between EDMD and the Koopman operator, a metric has been proposed for evaluating the fidelity of EDMD results, without assuming access to the analytical "true" eigendata [2].

Applications to Complex Systems

EDMD has been successfully applied to systems with simple dynamics such as fixed points or limit cycles. For example, it has been applied to flow past a circular cylinder as it transitions to a limit cycle of vortex shedding [3]. However, EDMD has not been validated for use on more complicated systems like turbulent flow. To that end, my work so far has been in applying EDMD to model systems with some of the same complex features as turbulence, namely ergodicity, mixing, and chaos.

We have found that for ergodic systems with pure point spectrum, EDMD (with reasonable choices for observables) correctly identifies the Koopman eigenvalues and eigenfunctions associated with the transformation, and the recently proposed error metric (with no access to analytical results) correctly indicates their validity. For mixing systems, which have only continuous spectrum, EDMD does not correctly identify the spectrum, and the error metric correctly indicates that they are useless. For systems with a "structured" component such as point spectrum and a "random" component such as continuous spectrum, EDMD can still correctly identify the point spectrum, and with the proposed error metric we can distinguish the "structured" and "random" components from each other.

Future work in this area will bring us closer to understanding how EDMD applies to turbulent systems. Some theoretical results to back up the findings on model systems will be found, and systems of increased complexity will be studied.

Turbulent Separation Bubbles

Separation bubbles, where the boundary layer detaches from a wall and reattaches downstream, are present in a variety of applications from airfoils at high angle of attack to corners, and often in cases of turbulent flow. They can arise from geometric discontinuities like a backward-facing step or from adverse pressure gradients. These turbulent separation bubbles have long been studied experimentally [4][5] and simulated numerically [6]. However, an observed low-frequency oscillation mode (Strouhal number approximately 0.01), typically called the "breathing" mode, has defied explanation, with numerous proposed causal mechanisms never quite matching all the data nor being conclusively proven [5][7][8].

In a planned collaboration between Cattafesta, Mittal, Meneveau, and Rowley, researchers at Florida State University will perform experiments involving a turbulent separation bubble induced by an adverse pressure gradient. Unlike previous experiments, this will include perturbations to the flow so that hopefully the causes, dynamics, and scaling relationships can be determined. Simulations will be performed at Johns Hopkins University as well. EDMD will be used at Princeton to help interpret the results.

Using EDMD, relevant features of the observed flow can be extracted, such as modes that oscillate at relevant frequencies. The physical structure of these modes should provide clues about the causes of oscillations. Also, producing a phenomenologically accurate reduced order model of turbulent separation bubble behavior could be useful for future control efforts. Extending EDMD to accurately capture mixing could apply here and in many other turbulent systems.

References

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