Koopman Operator and Nonlinear Dynamical Systems With Input

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Linear control theory is a well established field that provides us with a variety of tools for analyzing linear dynamical systems. Convenient methods exist for checking stability and controllability of linear systems, formulating control laws, and determining effects of model uncertainties and external disturbances. Moreover, the linear theory can be used in conjunction with Model Reduction techniques (such Proper Orthogonal Decomposition) for controlling systems of large dimensionality such as non-steady flow fields in fluids[1].

Non-linear dynamical systems can sometimes be linearized about fixed point and trajectories. However, resulting linearized systems are inaccurate away from their nominal trajectories and cannot capture some intrinsically non-linear phenomena. In particular, limit cycles do not appear in a linear systems although they are common in many physical setups such as low Reynolds flows[2]. Unfortunately, the tools for studying general nonlinear systems are very limited compared to the linear control theory.

We are therefore motivated to find a change of coordinates that transforms non-linear systems into equivalent linear systems. In the case of autonomous systems (i.e. without control input), we have shown that such coordinates are related to the Koopman operator. To be precise, the "desired" coordinates are linear combinations of Koopman eigenfunctions which, by their definition, evolve linearly in time.

Koopman eigenfunctions can be numerically approximated from given trajectories of the system via data driven approaches such as Extended Dynamic Mode Decomposition (EDMD)[3], and from the Taylor expansion of system dynamics around fixed points[4]. The later approach, happens do be related to the Normal form of the system and its Carleman linearization. In particular, we have shown that analytic Koopman eigenfunctions exist if and only if the Normal form of the system has no non-linear terms.

While the autonomous evolution of Koopman eigenfunctions is always linear, the situation for non-linear systems with input in more subtle. Considering a general n dimensional system, we have formulated $n-1$ necessary conditions on the input terms, for the system to appear linear in "Koopman Coordinates". The large amount of constraints suggests that one should not expect to find a transformation to bring an arbitrary non-linear system into a linear form. Instead, we will limit our focus to specific non-linear systems and their approximations.

In future work, we will treat systems of special forms, e.g. cases in which the system is linear in Koopman coordinates, or choose to satisfy the constraints partially, e.g. by choosing to control only the “slow” modes when appropriate. We will also consider the physical meaning of Koopman eigenfunctions in specific applications such as reduced order models of low Reynolds flows, and in particular address the question: How do linear control terms in Koopman coordinates appear in the original system? Additionally, we will augment model reduction and Koopman related algorithms with control terms to allow us to study the effects of feedback on Koopman eigenfunctions from a numerical perspective.

References

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